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# Comparison of the Efficiencies of Class D and Class E Rectifiers

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**Abstract** - The efficiencies of the Class D and Class E rectifiers are compared considering power losses in the parasitic resistances of capacitors, inductors and transformer windings, and conduction losses in the diodes. Equations for efficiency are plotted as functions of the output current with an output voltage  $V_O = 5$  V and a maximum output power  $P_O = 100$  W. It is shown that center-tapped rectifiers are the most suitable for low-voltage applications. Class E rectifier efficiency is higher than that of Class D circuit in the higher range of the output current. The Class E circuit is more suitable for low-current high-frequency applications. The results of this paper can be used by designers in selecting circuits with a better efficiency with respect to their specific load requirements.

## I. INTRODUCTION

Resonant dc-dc converters represent the most promising solution to the problem of size and weight reduction of off-line power supplies. The inverter section of the converter contains a resonant circuit at the output, operated as a sinusoidal current or voltage source [1]-[5]. The dc-dc power conversion is achieved by rectifying these waveforms. Power supplies for VLSI circuits must operate at a high-efficiency with a low-voltage, high-current output, e.g., 3.3 V or 5 V and 10 A. If off-line power supplies are used, the inverter section has a high-efficiency, e.g., 95 % [2]-[5]. Therefore, the total efficiency of the converter is mostly affected by the rectifier, where high currents flow through the circuit components. Because of this, one of the major problems facing designers is the rectifier topology which optimizes the converter efficiency.

The purpose of this paper is to compare the efficiencies of Class D rectifiers [2]-[4] [6] and Class E rectifiers [7]-[9] and determine which circuits are more suitable for applications in converters for VLSI circuits. The equations of rectifier efficiencies were determined as functions of the output current taking into account the parasitic resistances of diodes, capacitors, inductors, transformer windings, as well as diode offset voltages. The significance of the paper is that the presented method allows the designers to select the rectifier circuit which can be operated with higher efficiency, once specific applications are given.

## II. DETERMINATION OF THE RECTIFIER EFFICIENCIES

The analyses of rectifiers are carried out under the following assumptions:

1) The diode in the ON state is modelled by a series combination of a constant-voltage battery  $V_F$  and a constant resistance  $R_F$ , where  $V_F$  represents the diode threshold voltage and  $R_F$  diode forward resistance. The diode in the OFF state is modelled by an open switch. According to this assumption, the conduction loss in a diode is expressed as follows

$$P_D = V_F I_{D(AV)} + R_F I_{Drms}^2 \quad (1)$$

where  $I_{D(AV)}$  and  $I_{Drms}$  are the average and rms values of the current through the diode, respectively.

2) The voltage-driven rectifiers are driven by an ideal sinusoidal voltage source and the amplitude of the input voltage  $V_{Rim}$  is much higher than  $V_F$ . The current-driven rectifiers are driven by an ideal sinusoidal current source.

4) Filter inductance  $L_f$  in the voltage-driven rectifiers is large enough that the current ripple is negligible with respect to the dc output current  $I_O$ . Inductor  $L_f$  equivalent resistance is  $r_f$ .

5) Core losses in the transformer are assumed to be much smaller than copper losses. Copper losses are evaluated considering the equivalent resistance of the windings as it is seen at the terminals of the secondary winding  $r_{Cu}$ .

6) Losses in the capacitors are evaluated considering their equivalent series resistances. The equivalent series resistance of the filter capacitor is  $R_{ESRf}$  and equivalent series resistance of the parallel capacitor in the Class E rectifier is  $R_{ESR}$ .

Fig. 1 shows the Class D current-driven rectifier circuits, including the insulation transformer. In the half-wave rectifier, the average and the rms values of the current through each diode are  $I_{D(AV)} = I_O$  and  $I_{Drms} = \pi I_O / 2$ , respectively. The rms value of the current through the filter capacitor is  $I_{C(AV)} = I_O$  and  $I_{Crms} = \pi I_O / 2$ . The rms value of the current through the secondary winding of the transformer and blocking capacitor  $C_c$  is  $I_{Curms} = \pi I_O / \sqrt{2}$ . Since the output power delivered to load resistance  $R_L$  is  $P_O = R_L I_O^2$ , the rectifier efficiency is

$$\eta_{DCHW} = \left[ 1 + \frac{2V_F}{V_O} + \frac{\pi^2 R_F}{2R_L} + \frac{r_{ESRf}}{R_L} \left( \frac{\pi^2}{4} - 1 \right) + \frac{\pi^2 r_{Cu}}{2R_L} + \frac{\pi^2 r_{ESR1}}{2R_L} \right]^{-1} \quad (2)$$

In the Class D, current-driven, center-tapped rectifier, the average and rms values of the current through each diode modify as  $I_{D(AV)} = I_O / 2$  and  $I_{Drms} = \pi I_O / 4$ , respectively. Moreover, the rms current through the filter capacitor is

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$I_{Cfrms} = I_O(\pi^2/8-1)^{1/2}$  and through the transformer secondary winding is  $I_{Curms} = \pi I_O/4$ . Since an equivalent resistance  $r_{Cu}/2$  in series with each secondary winding has been considered, the final expression for the efficiency of the Class D, current-driven rectifier is

$$\eta_{DCCT} = \left[ 1 + \frac{V_F}{V_o} + \frac{\pi^2 R_F}{8 R_L} + \frac{r_{ESR}}{R_L} \left( \frac{\pi^2}{8} - 1 \right) + \frac{\pi^2 r_{Cu}}{16 R_L} \right]^{-1} \quad (3)$$

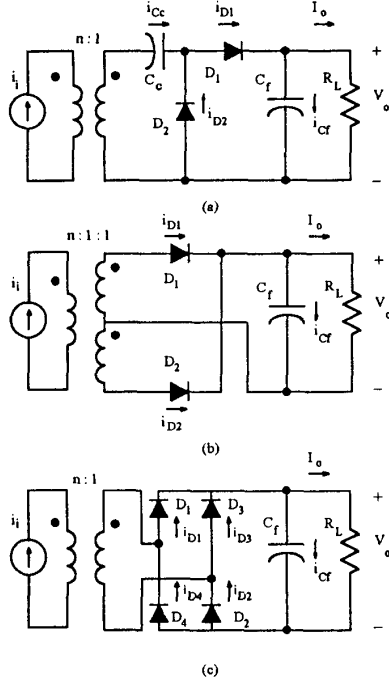


Fig. 1. Current-driven Class D rectifiers: (a) Half-wave circuit. (b) Center-tapped circuit. (c) Full Bridge circuit.

In the Class D, current-driven, full-bridge rectifier, conduction losses in each diode and in the filter capacitor are the same as in center-tapped rectifier. Since two diodes conduct simultaneously, the overall power loss in diode parasitics is doubled. The transformer power loss is twice that in the center-tapped rectifier. As a result, the expression for the full-bridge rectifier efficiency becomes

$$\eta_{DCFB} = \left[ 1 + \frac{2V_F}{V_o} + \frac{\pi^2 R_F}{4 R_L} + \frac{r_{ESR}}{R_L} \left( \frac{\pi^2}{8} - 1 \right) + \frac{\pi^2 r_{Cu}}{8 R_L} \right]^{-1} \quad (4)$$

The circuits of the Class D, voltage-driven rectifiers are shown in Fig. 2. According to assumption (4), the input current of the Class D, voltage-driven rectifiers is a square wave, varying from  $-I_O/2$  and  $I_O/2$ . Because of this, the average value of the current through each diode is  $I_{D(AV)} = I_O/2$  and its rms value is  $I_{Drms} = I_O/\sqrt{2}$ . The rms value of the current through inductor  $L_f$  and the transformer secondary winding are  $I_{Lfrms} = I_O$  and  $I_{Curms} = I_O/2$ , respectively. The rms value of the current in the filter capacitor ESR is  $I_{Cfrms} = 0.2346 I_O R_L / f L_f$ . As

a result, the expression for the efficiency of the Class D, voltage-driven, half-wave rectifier is given by

$$\eta_{DVCT} = \left[ 1 + \frac{V_F}{V_o} + \frac{R_F}{R_L} + \frac{r_f}{R_L} + \frac{0.2346^2 r_{ESR} R_L}{f^2 L_f^2} + \frac{r_{Cu}}{4 R_L} \right]^{-1} \quad (5)$$

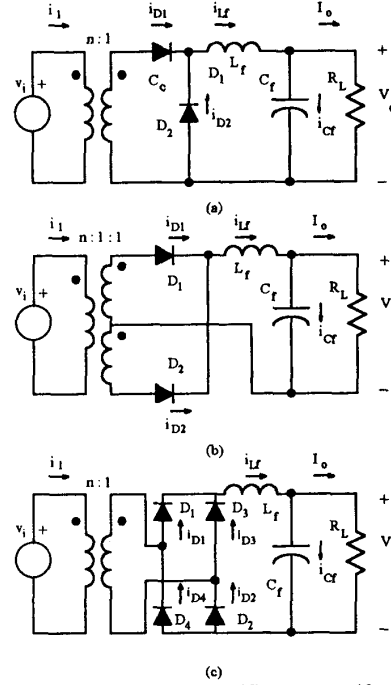


Fig. 2. Voltage-driven Class D rectifiers: (a) Half-wave circuit. (b) Center-tapped circuit. (c) Full-bridge circuit.

In the Class D, voltage-driven, center-tapped rectifier of Fig. 2(b), the output current  $I_O$  flows through diodes  $D_1$  and  $D_2$  for half a period. Consequently, the average and rms values of the current through each diode are as in the half-wave rectifier. The rms current through the filter capacitor is  $I_{Cfrms} = 0.0337 I_O R_L / f L_f$ . The rms value of the current through the secondary winding is equal to the output current  $I_O$ . Because of this, the efficiency of the Class D, center-tapped, voltage-driven rectifier is expressed as

$$\eta_{DVCT} = \left[ 1 + \frac{V_F}{V_o} + \frac{R_F}{R_L} + \frac{r_f}{R_L} + \frac{0.0337^2 r_{ESR} R_L}{f^2 L_f^2} + \frac{r_{Cu}}{R_L} \right]^{-1} \quad (6)$$

The rms values of the currents through the Class D, full-bridge, voltage-driven rectifier of Fig. 2(c) are the same as those evaluated for the center-tapped, voltage-driven rectifier. Since two diodes conduct simultaneously in the full-bridge rectifier, the expression for its efficiency is as follows

$$\eta_{DVFB} = \left[ 1 + \frac{2V_F}{V_o} + \frac{2R_F}{R_L} + \frac{r_f}{R_L} + \frac{0.0337^2 r_{ESR} R_L}{f^2 L_f^2} + \frac{r_{Cu}}{R_L} \right]^{-1} \quad (7)$$

Fig. 3(a) shows the circuits of the Class E, current-driven, half-wave rectifier. The rms value of the current through diodes  $D_1$  is given by

$$I_{D_{rms}} = I_o \left[ D + \frac{2\pi D + \sin\phi \cos\phi}{4\pi \sin^2(\phi + 2\pi D)} + \frac{3}{4\pi \tan(\phi + 2\pi D)} - \frac{\cos\phi}{\pi \sin(\phi + 2\pi D)} \right]^{1/2} \quad (8)$$

where  $\phi$  is the turn-on delay angle of diode  $D_1$  and  $D$  is its on-duty cycle. The relationships among delay angle  $\phi$ , load resistance  $R_L$ , and duty cycle  $D$  are given in [9]. Similarly, the rms value of the current through the parallel capacitor  $C_f$  is

$$I_{C_{rms}} = I_o \left[ 1 - D + \frac{2\pi(1-D) - \sin\phi \cos\phi}{4\pi \sin^2(\phi + 2\pi D)} - \frac{3}{4\pi \tan(\phi + 2\pi D)} + \frac{\cos\phi}{\pi \sin(\phi + 2\pi D)} \right]^{1/2} \quad (9)$$

The current through the secondary winding of the transformer and filter capacitor  $C_f$  is equal to the current of the source, that is, the ac component of  $ni$ . As a result, we have

$$I_{C_{rms}} = I_{C_{fms}} = \frac{I_o}{\sqrt{2} \sin(\phi + 2\pi D)} \quad (10)$$

Considering an equivalent resistance  $r_{Cu}/2$  in series with each secondary winding, the efficiency of the Class E, current-driven, half-wave rectifier is determined as follows

$$\eta_{ECHW} = \left\{ 1 + \frac{V_F}{V_o} + \frac{R_F}{R_L} \left[ D + \frac{2\pi D + \sin\phi \cos\phi}{4\pi \sin^2(\phi + 2\pi D)} + \frac{3}{4\pi \tan(\phi + 2\pi D)} - \frac{\cos\phi}{\pi \sin(\phi + 2\pi D)} \right] + \frac{r_{ESR}}{R_L} \left[ 1 - D + \frac{2\pi(1-D) - \sin\phi \cos\phi}{4\pi \sin^2(\phi + 2\pi D)} - \frac{3}{4\pi \tan(\phi + 2\pi D)} + \frac{\cos\phi}{\pi \sin(\phi + 2\pi D)} \right] + \frac{r_{Cu}}{2R_L \sin^2(\phi + 2\pi D)} + \frac{r_{ESRf}}{2R_L \sin^2(\phi + 2\pi D)} \right\}^{-1} \quad (11)$$

In the Class E, current-driven, center-tapped rectifier of Fig. 3(b), the amplitude of the current through each diode is one half of that flowing through the diode of the half-wave rectifier. Therefore, the conduction losses in each diode due to the forward voltage drop  $V_F$  are one half and the loss due to the forward resistance  $r_F$  is four times lower than that in the diode of the Class E half-wave rectifier. Also the power losses in parallel capacitors  $C_1$  and  $C_2$  and in the transformer windings are reduced by a factor of four. Moreover, in the Class E center-tapped rectifier, the current through filter capacitor  $C_f$  is nearly zero. This results in no loss operation of capacitor  $C_f$ . Because all of this, the efficiency of the Class E, current-driven, center-tapped rectifier is expressed as

$$\eta_{ECT} = \left\{ 1 + \frac{V_F}{V_o} + \frac{R_F}{2R_L} \left[ D + \frac{2\pi D + \sin\phi \cos\phi}{4\pi \sin^2(\phi + 2\pi D)} + \frac{3}{4\pi \tan(\phi + 2\pi D)} - \frac{\cos\phi}{\pi \sin(\phi + 2\pi D)} \right] + \frac{r_{ESR}}{2R_L} \left[ 1 - D + \frac{2\pi(1-D) - \sin\phi \cos\phi}{4\pi \sin^2(\phi + 2\pi D)} - \frac{3}{4\pi \tan(\phi + 2\pi D)} + \frac{\cos\phi}{\pi \sin(\phi + 2\pi D)} \right] + \frac{r_{Cu}}{8R_L \sin^2(\phi + 2\pi D)} \right\}^{-1} \quad (12)$$

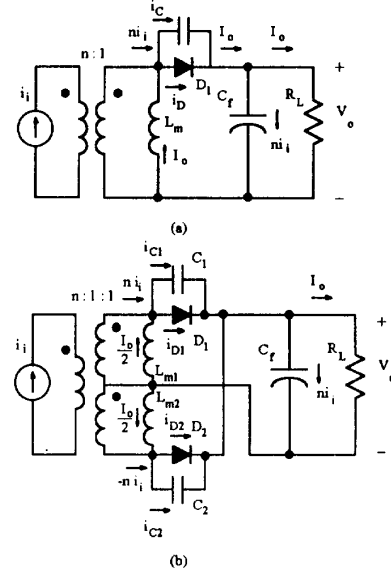


Fig. 3. Current-driven Class E rectifiers: (a) Half-wave circuit. (b) Center-tapped circuit.

The efficiencies of the Class D and Class E rectifiers are plotted in Fig. 4 as functions of the output current assuming a 5 V constant output voltage operation. According to the data measured while testing a 1 MHz, 100 W, Class E, center-tapped rectifier, the values of the components were chosen as follows:  $V_F = 0.3$  V,  $R_F = 0.033$   $\Omega$ ,  $R_{ESRf} = 0.04$   $\Omega$ ,  $r_f = 0.125$   $\Omega$ ,  $L_F = 5$   $\mu$ H,  $r_{Cu} = 0.01$   $\Omega$ , and  $D_{MAX} = 0.45$ . The efficiencies of the Class E rectifiers are plotted for  $r_{ESR} = 0.01$   $\Omega$  and  $r_{ESR} = 0.04$   $\Omega$ . All Class D rectifiers have high efficiencies at low loads while the Class E, half-wave circuit has a poor efficiency over the entire load range even if  $r_{ESR}$  is low. Since only the current-driven, center-tapped Class D and Class E rectifiers have an efficiency greater than 78 % over the entire load range, they result suitable for low output voltage applications. The efficiency of Class D rectifier decreases from 94 % to 85 % for an output current increasing from 0 to 12 A and decreases to 78 % at the full load. The Class E circuit efficiency slightly decreases from 88 % to 83 % for  $I_o$  increasing from 10 % to 100 % of the maximum value.

As shown in Fig. 5, Class D rectifier has a better performance in low current applications, i.e.,  $I_o < 10$  A. The experimental waveform of the diode voltage depicted in Fig. 6 demonstrates that the Class E rectifier is not affected by switching losses and parasitic oscillations occurring at high-frequencies. Therefore, it represents an appropriate solution for high-power density, high-output current applications in the megahertz range.

### III. CONCLUSIONS

The efficiencies of Class D, current-driven and voltage-driven rectifiers, along with those of Class E current-driven rectifiers have been analytically evaluated and plotted

versus the dc load current  $I_O$ , assuming a constant output voltage  $V_O = 5$  V operation of the rectifiers. Circuit parameters have been chosen according to those measured in experimentally testing a 100 W, 1 MHz Class E, current-driven, center-tapped rectifier with an output voltage  $V_O = 5$  V. Only the Class D and Class E, current-driven, center-tapped rectifiers operate with high efficiencies over the entire load range. The Class E rectifier has a higher efficiency for a high load current and has zero switching losses. Therefore, it is suitable for high-frequency, and high output current applications. The presented method can be applied to other rectifier circuits to select the rectifier operating with the higher efficiency for any given application. Because of this, this paper represents a helpful tool in the optimization of the efficiency of power converters.

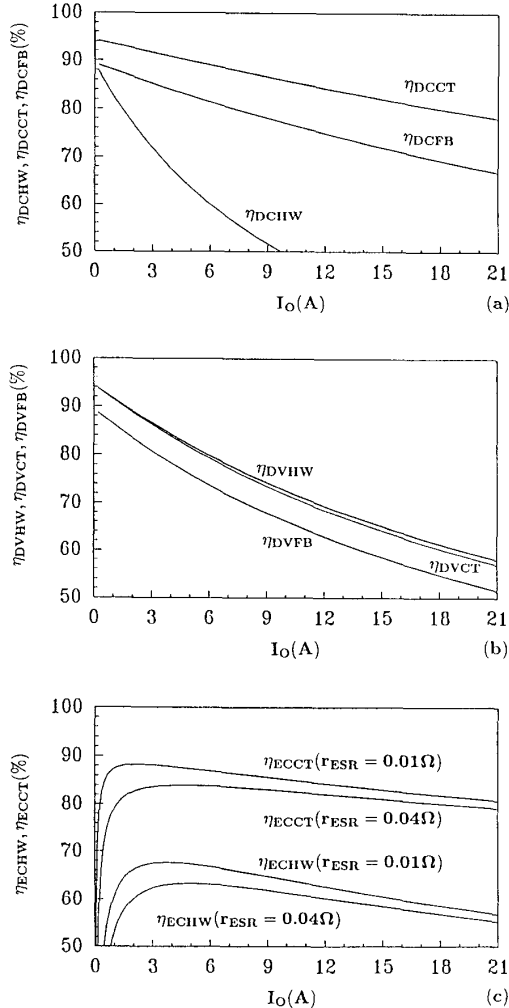


Fig. 4. Efficiency of Class D and Class E rectifiers as functions of the output current  $I_O$ . (a) Efficiency of Class D, current-driven rectifiers. (b) Efficiency of Class D, voltage-driven rectifiers. (c) Efficiency of Class E, current-driven rectifiers.

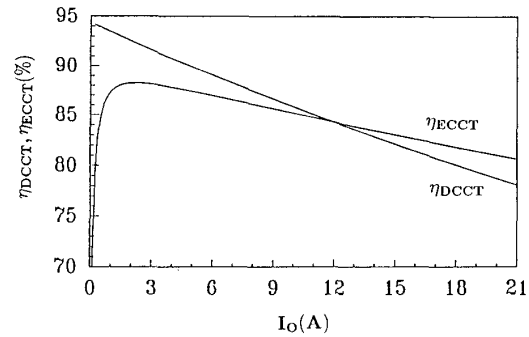


Fig. 5. Efficiency of Class D current-driven, center-tapped rectifier  $\eta_{DCCT}$  and efficiency of Class E, current-driven, center-tapped rectifier  $\eta_{ECCT}$  as functions of output current  $I_O$ .

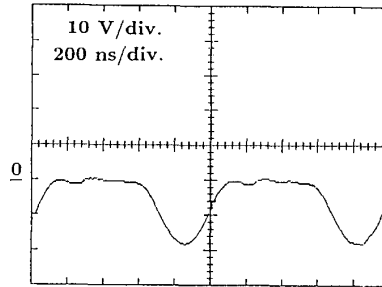


Fig. 6. Experimental waveform of the voltage across one diode of Class E, current-driven, center-tapped rectifier measured at the frequency  $f = 1$  MHz, with an output voltage  $V_O = 5$  V, and an output current  $I_O = 5$  A.

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